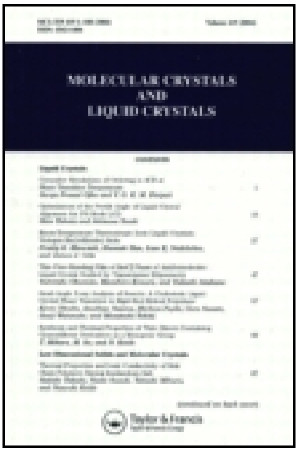

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Optical Transmission Factor of a Ferronematic Liquid Crystal Under Magnetic Field Induced Orientational Transitions

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In the framework of continuum theory we study the light transmission factor of a ferronematic liquid crystal as a function of external magnetic field, the anchoring energy with the boundary of a layer, and material parameters of a ferronematic. We show that magnetic field induced Fréedericksz transition in ferronematics in twist geometry can be either a first- or second-order phase transition depending on the anchoring energy between the director and magnetic particles, i.e., it exhibits the tricritical behavior. The light transmission factor for first- or second-order transition is also studied.

Keywords Ferronematic; magnetic suspension; optical transmission; orientational transitions

Introduction

Ferronematics represent suspensions of needle-like magnetic particles on the basis of nematic liquid crystals and are optically uniaxial media. It is known that the orientation of optical axis (the so-called director $\boldsymbol{n}$) of a ferronematic in a magnetic field is determined by the diamagnetic interaction of nematic molecules with a field, and also by the influence of ferromagnetic interaction of the particles added in a liquid crystal with a magnetic field.

We consider the layer of a ferronematic, sandwiched between two parallel flat surfaces. The superficial energy of interaction of a ferronematic with these surfaces looks like

$$F_S = \frac{1}{2} W_0 \sin^2\theta(1 - \zeta \sin^2\theta), \tag{1}$$

where θ is the angle between the axis of easy orientation $\boldsymbol{e} = (1,\ 0,\ 0)$ and the director $\mathbf{n}$, W_0 is the surface density of coupling energy of liquid crystal molecules with the boundaries of a cell, $\zeta \in [-1, 1]$ is the additional parameter of the surface coupling energy. For $W_0 > 0$ the potential (1) depending on the values of parameter ζ has either one or two minima. The potential (1) was used earlier in [1–5] in which it has been shown, that for $\zeta > 0$ in nematics and suspensions on their basis there are magnetic field induced Freedericksz transitions of the first order, while for $\zeta \leq 0$ – the second order transitions.

The addition of a small amount of magnetic particles in nematic liquid crystal does not worsen its optical characteristics; on the contrary, it promotes easier reorientation of its

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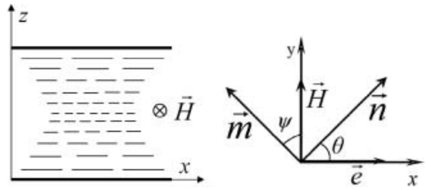

Figure 1. Geometry.

optical axis in a magnetic field. Let the homeotropic coupling of liquid crystal molecules be created on the surface of magnetic particles, and the ferronematic possesses positive anisotropy of diamagnetic susceptibility. The magnetic field, directed parallel to the surfaces of a layer across an axis of easy orientation, leads to the twisting of the director in a cell (see Fig. 1). If linearly polarized along $\boldsymbol{e}$ light wave falls perpendicularly to the boundaries of a layer, i.e. along the twisting axis of a director, and then a ferronematic layer in a magnetic field can be considered as the anisotropic media with rotation. In any point of such media there are two waves (the ordinary and extraordinary waves), which are linearly polarized along "local" main axes. The directions of polarization of waves rotate according to the rotation of "local" main axes of a nematic with propagation of waves along the twisting axis. At the same time the difference of phases of these two waves remain the same, as in non twisted nematic environment. Such phenomenon is called the adiabatic tracking of polarization [6]; this optical property is interesting for use in nematic twist cells which rotate the plane of polarization of light wave [7].

In a ferronematic layer there are similar processes; however, the angles of rotation of the director and consequently, the plane of polarization are determined by the coupling conditions on the layer boundaries, the coupling energy between the magnetic particles and liquid crystal matrix, and external magnetic field. It is known that for light propagation through polarizers or polarizing interference filters its intensity can be reduced in some cases. The layer of a ferronematic with rotation of optical axis can be considered as such filter [6]. Therefore in the present work we study light transmission factor of a ferronematic as a function of material parameters of a ferronematic, the energy of surface coupling, and an external magnetic field.

Equilibrium State Equations

Let us obtain the set of equations that determine the equilibrium state of ferronematic [4, 5]. The equilibrium configuration of the director and the magnetization is determined from the condition of a minimum of full free energy of a ferronematic

$$F = \int F_V dV + \oint F_S dS, \tag{2}$$

including both volume and surface parts. The volume density of a ferronematic free energy is determined by the expression [8, 9]

$$\begin{aligned} F_V = {} & \frac{1}{2}\left[K_{11}(\nabla \cdot \boldsymbol{n})^2 + K_{22}(\boldsymbol{n} \cdot \nabla \times \boldsymbol{n})^2 + K_{33}(\boldsymbol{n} \times \nabla \times \boldsymbol{n})^2\right] \\ & -\frac{1}{2}\mu_0 \chi_a (\boldsymbol{nH})^2 - \mu_0 M_S f \boldsymbol{mH} + \frac{W_p}{d} f(\boldsymbol{mn})^2 + \frac{k_B T}{v} f \ln f. \end{aligned} \tag{3}$$

Here K_{ii} are constants of orientational elasticity of a liquid crystal; $\chi_a > 0$ is the anisotropy of a diamagnetic susceptibility; $\boldsymbol{H}$ is the magnetic field; M_S is the magnetization of saturation of a particles material; $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1} is the permeability of free space; v is the volume of a particle; f is the volume fraction of the particles in a suspension; $\boldsymbol{m}$ is the unit vector of a ferronematic magnetization $\boldsymbol{M} = M_s f \boldsymbol{m}$; d is the particle diameter; T is the temperature; k_B is the Boltzmann constant. The parameter W_p is the coupling energy of magnetic particles with a nematic matrix. We assume $W_p > 0$, that corresponds to soft homeotropic coupling of magnetic particles with a nematic matrix, at which $\boldsymbol{m} \perp \boldsymbol{n}$ in the absence of external fields.

Let us direct the magnetic field $\boldsymbol{H} = (0, H, 0)$ along the boundaries of a layer. In this case the components of $\boldsymbol{n}$ and $\boldsymbol{m}$ can be written as

$$\boldsymbol{n} = [\cos\theta(z), \sin\theta(z), 0],$$
$$\boldsymbol{m} = [-\sin\psi(z), \cos\psi(z), 0], \tag{4}$$

where $\theta(z)$ is the angle between the director and the x axis, and $\psi(z)$ is the angle between the magnetization and the field $\boldsymbol{H}$ (see Fig. 1).

Let us choose the thickness L of a cell as the unit of length and define the dimensionless coordinate $\tilde{z} = z/L$ and dimensionless parameters

$$\xi = M_S f_0 L \sqrt{\frac{\mu_0}{K_{22}\chi_a}}, \quad \kappa = \frac{L^2 k_B T f_0}{v K_{22}}, \quad w_p = \frac{L^2 f_0 W_p}{K_{22} d}, \quad w_0 = \frac{W_0 L}{K_{22}},$$

where we introduce the average concentration of magnetic particles in a suspension $f_0 = \mathcal{N} v/V$ ($\mathcal{N}$ is the number of magnetic particles, V is the volume of ferronematic), and the dimensionless magnetic field $h = LH\sqrt{\mu_0 \chi_a / K_{22}}$.

For the suspension of γ-Fe_2O_3 magnetic particles (SI units) with the magnetization of saturation $M_S = 3.4 \times 10^5$ A m^{-1}, with the diameter $d = 7 \times 10^{-8}$ m, volume $v = 2 \times 10^{-21}$ m^3, and average concentration of magnetic particles $f_0 = 10^{-6}$ [10] on the basis of nematic MBBA at temperature 22°C with $K_{22} = 3 \times 10^{-12}$ N, and $\chi_a = 1 \times 10^{-7}$ [11] for the cell thickness $L = 10^{-4}$ m, the dimensionless parameters ξ and κ are as follows $\xi \approx 20$ and $\kappa \approx 10^{-2}$.

The equations of orientational equilibrium one can obtain from the conditions of a minimum of full free energy (2) over $\varphi(z)$, $\psi(z)$, and $f(z)$:

$$\xi h \sin\psi = w_p \sin 2(\theta - \psi), \tag{5}$$

$$f = f_0 G \exp\left\{\frac{\xi h}{\kappa}\cos\psi - \frac{w_p}{\kappa}\sin^2(\theta - \psi)\right\}, \tag{6}$$

$$\tilde{z} = \int_{\theta_0}^{\theta(\tilde{z})} A^{-1/2}(\theta, \psi(\theta)) d\theta, \tag{7}$$

$$A^{1/2}(\theta_0, \psi(\theta_0)) = \tfrac{1}{2} w_0 \sin 2\theta_0 \left[1 - 2\zeta \sin^2\theta_0\right], \tag{8}$$

$$\int_{\theta_0}^{\theta_m} A^{-1/2}(\theta, \psi(\theta)) d\theta = \tfrac{1}{2}, \tag{9}$$

where the following notations are introduced

$$\theta_0 = \theta(\tilde{z})|_{\tilde{z}=0}, \quad \theta_m = \theta(\tilde{z})|_{\tilde{z}=1/2},$$
$$A(\theta, \psi(\theta)) = h^2\left(\sin^2\theta_m - \sin^2\theta\right) + 2\kappa[f(\theta_m) - f(\theta)]/f_0.$$

Here the quantity G is determined from the condition $\int f\,dV = \mathcal{N}v$ of constancy of particles number in a suspension.

Eqs. (4)–(9) have three types of solutions which corresponds to three ferronematic phases with various types of ordering [4]:

- the uniform phase $\theta = \psi = 0$ corresponds to the initial state of a ferronematic, for which the director is oriented along the axis of easy orientation, and magnetic particles are perpendicular to the director (the so-called homeotropic anchoring between the magnetic particles and the director). This phase can exist for $h \le h_F$, where h_F is the Freedericksz field determined from the equations [4]

$$\lambda \tan\lambda/2 = w_0, \quad \lambda^2 = h_F^2 - \frac{2w_p\xi h_F}{2w_p + \xi h_F}. \tag{10}$$

- the distorted phase of a ferronematic $0 < \theta(\tilde{z}) < \pi/2$, which can be stable for $h_F \le h \le h_S$, where the threshold field h_S is determined from the equations [4]

$$\alpha \tanh\alpha/2 = w_0(1 - 2\zeta), \quad \alpha^2 = h_S^2 + \frac{2w_p\xi h_S}{2w_p - \xi h_S}. \tag{11}$$

- the saturation state $\theta = \pi/2$, $\psi = 0$, for which both the director, and the magnetization are aligned along the field direction. It is possible for $h \ge h_S$.

Jones Method for Calculation of Light Transmission Factor in a Ferronematic

As it was already mentioned above, if linearly polarized light wave falls on ferronematic cell perpendicularly to the boundaries, the polarization plane of a light wave in a layer of a ferronematic will rotate according to the director rotation under the magnetic field action. Let the ferronematic cell, which is twisted by a magnetic field, be placed between two parallel polarizers, transmission axes of which are parallel to the x axis. Let us determine light transmission factor propagated through such optical system.

Such a problem can be solved by the Jones method of calculation of optical systems (see, for example, [12–14]). According to this method, each elliptically polarized oscillation can be represented by two-component column vector (Jones vector) which components are complex amplitudes of electric field of a light wave. Let the state of polarization of light wave falling on a layer is described by Jones vector

$$\boldsymbol{J}_0 = \begin{bmatrix} E_x \\ E_y \end{bmatrix},$$

and the polarization state of the transmitted wave is described with

$$\boldsymbol{J}' = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix},$$

where $E_x = |E_x|e^{i\delta_x}$, $E_y = |E_y|e^{i\delta_y}$, $E'_x = |E'_x|e^{i\delta'_x}$, and $E'_y = |E'_y|e^{i\delta'_y}$. Here E_x, E_y, E'_x, E'_y are amplitudes, and $\delta_x, \delta_y, \delta'_x, \delta'_y$ are initial phases of linear oscillations of electric field of incident and transmitted waves along the x and y axes, respectively. Then the transmission factor on intensity of light is defined by the expression [12, 13]

$$\tau = \frac{|E'_x|^2 + |E'_y|^2}{|E_x|^2 + |E_y|^2}. \tag{12}$$

The interaction of incident wave with the ferronematic can be described by the expression

$$\boldsymbol{J}' = Q\boldsymbol{J}_0. \tag{13}$$

Here Q is 2×2 matrix, which is called the Jones matrix of phase delay (lag); it determines the influence of optical system on the incident wave [13, 14]. Thus, for the determination of Jones vector $\boldsymbol{J}'$ for the transmitted wave it is necessary to construct the Jones matrix Q.

When the light is transmitted through the first polarizer, which has the transmission axis parallel to the x axis, the polarization of the wave is characterized by following Jones vector

$$\boldsymbol{J}_0 = \begin{bmatrix} E_x \\ 0 \end{bmatrix}. \tag{14}$$

This vector describes the linearly polarized wave which electric vector makes simple harmonious oscillation along the x axis with zero initial phase and amplitude E_x [13].

If the ferronematic is in the distorted phase and is limited by two surfaces, it is possible to divide it into the large number of very thin layers with the thicknesses $L/(2N)$, where $2N$ is the total number of the layers in a cell. Each layer represents the two-refractive plate revolved on some small angle $(\theta_m - \theta_0)/N$ concerning the previous one. We consider the bottom half of cell. The first thin layer is directed along the angle $\theta_0 + \varphi/N$ relative to the x axis, the second along the angle $\theta_0 + 2\varphi/N$, etc. In the middle of a ferronematic cell N-layer revolve on the maximal angle of rotation θ_m. The designation $\varphi = \theta_m - \theta_0$ is hereinafter introduced. If light falls normally to the layers, i.e. along the z axis, the Jones matrix of a phase delay for the first thin layer looks like

$$Q_1 = R(-\theta_0 - \varphi/N)Q_0R(\theta_0 + \varphi/N),$$

where

$$R\Big(\theta_0 + \frac{\varphi}{N}\Big) = \begin{bmatrix} \cos\left(\theta_0 + \frac{\varphi}{N}\right) & \sin\left(\theta_0 + \frac{\varphi}{N}\right) \\ -\sin\left(\theta_0 + \frac{\varphi}{N}\right) & \cos\left(\theta_0 + \frac{\varphi}{N}\right) \end{bmatrix}$$

is the rotation matrix of reference frame xyz by the angle $\theta_0 + \varphi/N$,

$$Q_0 = \begin{bmatrix} \exp\left\{-\frac{\Gamma}{2N}i\right\} & 0 \\ 0 & \exp\left\{-\frac{\Gamma}{2N}i\right\} \end{bmatrix}$$

is the matrix of phase delay of one layer concerning its main optical axes [6, 13]. Here $\Gamma = \pi L(n_e - n_o)/\lambda$ is the phase delay of half of cell of a ferronematic in the absence of twist (i.e. in the uniform state and in the saturation state), λ is the wave length of incident light, n_o и n_e is the ordinary and extraordinary refraction indices of a ferronematic [15].

The Jones matrixes for the second, k-th and N-th layer can be written down as

$$\begin{aligned} Q_2 &= R\left(-\theta_0 - \frac{2\varphi}{N}\right) Q_0 R\left(\theta_0 + \frac{2\varphi}{N}\right), \ldots, \\ Q_k &= R\left(-\theta_0 - \frac{\varphi}{N}k\right) Q_0 R\left(\theta_0 + \frac{\varphi}{N}k\right), \ldots, \\ Q_N &= R(-\theta_m) Q_0 R(\theta_m), \end{aligned}$$

where R is the rotation matrix on corresponding angles.

At the top part of a ferronematic cell the director rotates in the opposite side, i.e. the angles of orientation of thin layers vary from θ_m to θ_0. Having carried out similar consideration for the top part of a cell, we obtain Jones matrixes for $(N+1)$-th, $(N+k)$-th and $2N$-th a layer

$$\begin{aligned} Q_{N+1} &= R\left(-\theta_m + \frac{\varphi}{N}\right) Q_0 R\left(\theta_m + \frac{\varphi}{N}\right), \ldots, \\ Q_{N+k} &= R\left(-\theta_m + \frac{\varphi}{N}k\right) Q_0 R\left(\theta_m - \frac{\varphi}{N}k\right), \ldots, \\ Q_{2N} &= R(-\theta_0) Q_0 R(\theta_0). \end{aligned}$$

The total matrix of phase delay for the twisted ferronematic takes the form

$$\begin{aligned} Q = \prod_{n=2N}^{1} Q_n = \prod_{k=N}^{1} R\left(-\theta_m + \frac{\varphi}{N}k\right) Q_0 R\left(\theta_m - \frac{\varphi}{N}k\right) \\ \times \prod_{k=N}^{1} R\left(-\theta_0 - \frac{\varphi}{N}k\right) Q_0 R\left(\theta_0 + \frac{\varphi}{N}k\right). \end{aligned} \tag{15}$$

For the simplification of Eq. (15) we use the following property of rotation matrixes [12]

$$R(\alpha)R(\beta) = R(\alpha + \beta). \tag{16}$$

and obtain

$$Q = R(-\theta_0)\left[Q_0 R\left(-\frac{\varphi}{N}\right)\right]^N \left[Q_0 R\left(\frac{\varphi}{N}\right)\right]^N Q_0 R(\theta_0).$$

Let us now define

$$M(\varphi) = \left[Q_0 R\left(\frac{\varphi}{N}\right)\right]^N.$$

In accordance with [6, 14], the matrix elements are as follows

$$M_{11} = (M_{22})^* = \cos\frac{\varphi}{N} \exp\left\{-\frac{\Gamma}{2N}i\right\} \frac{\sin N\eta}{\sin\eta} - \frac{\sin(N-1)\eta}{\sin\eta};$$

$$M_{12} = -(M_{21})^* = \sin\frac{\varphi}{N}\exp\left\{-\frac{\Gamma}{2N}i\right\}\frac{\sin N\eta}{\sin\eta},$$

where η can be determined from

$$\cos\eta = \cos\frac{\varphi}{N}\cos\frac{\Gamma}{2N}.$$

Here the asterisk denotes the complex conjugation.

For ferronematic cell with the thickness $L = 10^{-4}$ m, the diameter of magnetic particles $d \approx 7\cdot 10^{-8}$ m, and the size of nematic molecules of order of few nanometers, the thickness of the layers is very small ($\approx 10^{-7}$ m) and N is large number. Thus for $N \to \infty$, we have

$$Q = R(-\theta_0)M(-\varphi)M(\varphi)R(\theta_0),$$

$$\eta^2 = \left(\frac{\varphi}{N}\right)^2 + \left(\frac{\Gamma}{2N}\right)^2 \text{ or}$$

$$B^2 = (N\eta)^2 = \varphi^2 + (\Gamma/2)^2, \tag{17}$$

$$M(\varphi) = \begin{bmatrix} \cos B - i\dfrac{\Gamma}{2}\dfrac{\sin B}{B} & \varphi\dfrac{\sin B}{B} \\ -\varphi\dfrac{\sin B}{B} & \cos B + i\dfrac{\Gamma}{2}\dfrac{\sin B}{B} \end{bmatrix}. \tag{18}$$

With the help of Eqs. (13), (14), and (17) we obtain the Jones vector $\boldsymbol{J}'$ after light transmission through twisted ferronematic cell and the second polarizer. Thus the second polarizer allocates only x-component of this vector, and the transmission factor (12) is as follows

$$\tau = \frac{E'_x E'^*_x}{E_x E^*_x} = \frac{|E'_x|^2}{|E_x|^2}.$$

If the phase lag is much more than the rotation angle $\Gamma >> \varphi$, it follows from Eq. (17) that $B = \Gamma/2$, and Eq. (18) gives

$$M = \begin{bmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{bmatrix}.$$

Then the transmission factor becomes

$$\tau = 1 - \sin^2 2\theta_0 \sin^2\Gamma. \tag{19}$$

Eq. (19) coincides with the expression obtained in Ref. [15] for the light intensity for a nematic in a magnetic field in the case of twist deformation.

Light Transmission Factor at Orientational Transitions

For numerical calculations we choose $\xi = 10$, $\kappa = 2.2\times 10^{-3}$, $w_p = 0.05$, $w_0 = 10$, $n_o = 1.5443$, $n_e = 1.7582$ for a ferronematic on the basis of liquid crystal MBBA [16], and $\lambda = 632.8$ nm, which corresponds to He-Ne laser.

In Fig. 2 the angles θ_m, θ_0, and light transmission factor τ as functions of magnetic field are shown. Continuous curves in Fig. 2 and following figures correspond to stable states of ferronematic, and the dashed curves correspond to unstable states. As it is seen from Fig. 2, in the field absence ($h = 0$) the ferronematic is in the uniform phase ($\theta_0 = \theta_m = 0$), in which

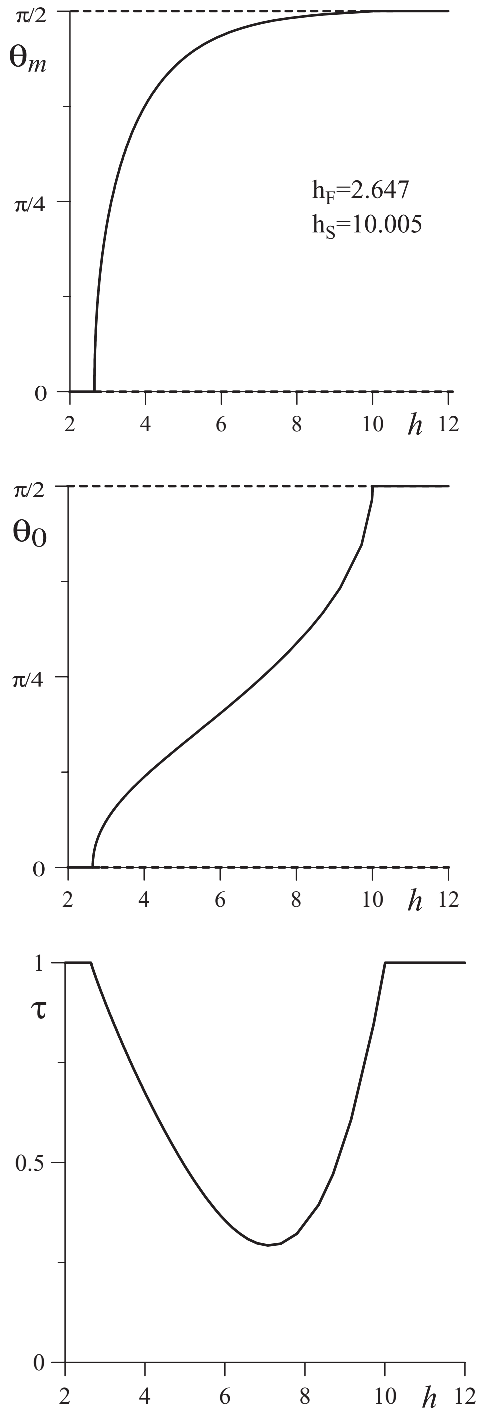

Figure 2. Dependencies of the angles θ_m, θ_0, and transmission factor τ on magnetic field h for $w_p = 0.05$ and $\zeta = 0$.

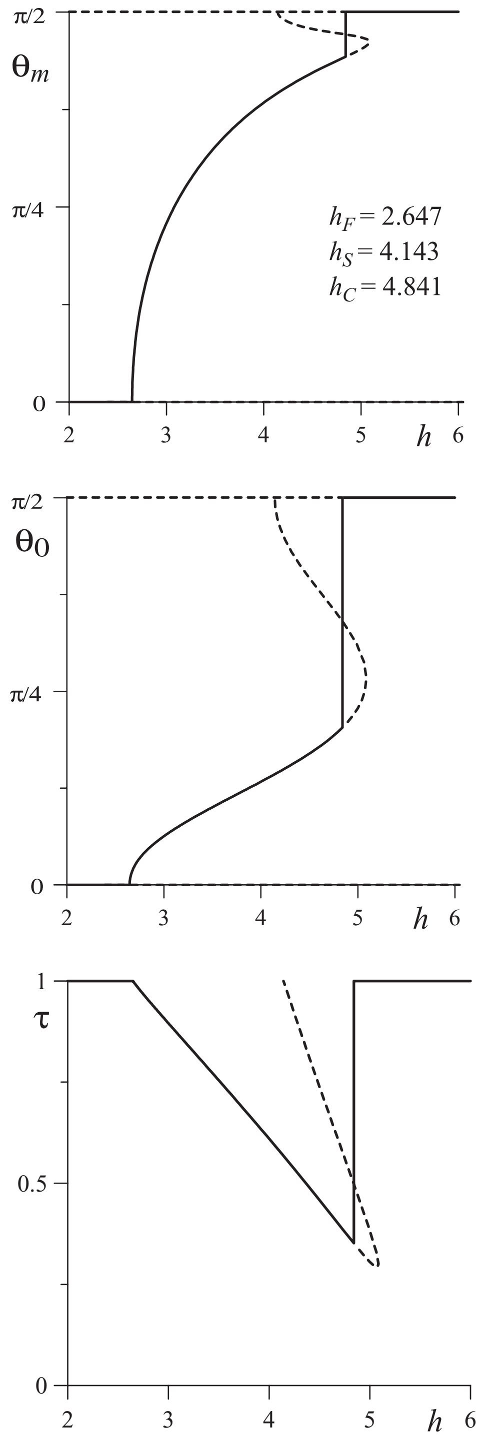

Figure 3. Dependencies of the angles θ_m, θ_0 and transmission factor τ on magnetic field h for $w_p = 0.05$ and $\zeta = 0.3$.

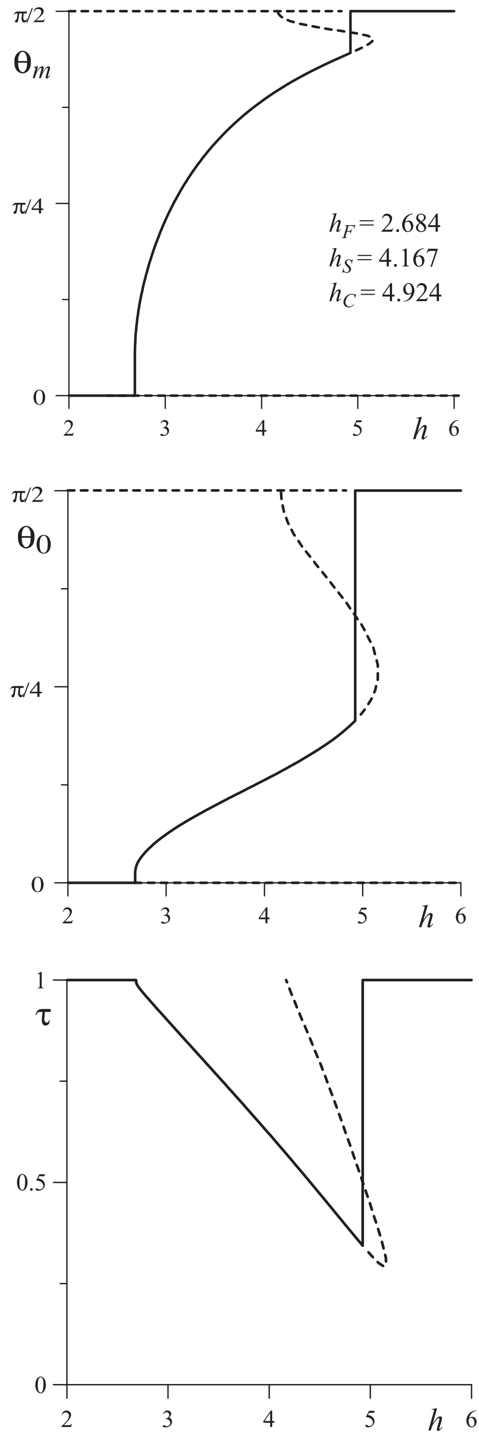

Figure 4. Dependencies of the angles θ_m, θ_0 and transmission factor τ on magnetic field h for $w_p = 0.1515$ and $\zeta = 0.3$.

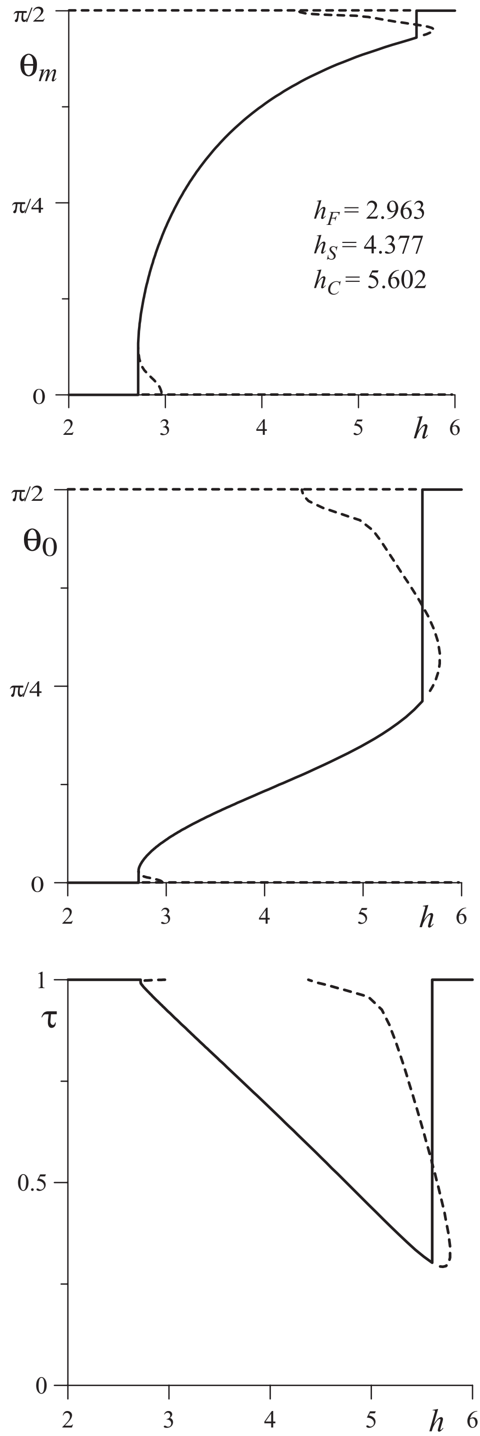

Figure 5. Dependencies of the angles θ_m, θ_0 and transmission factor τ on magnetic field h for $w_p = 1$ and $\zeta = 0.3$.

the director is oriented along the easy orientation axis $\boldsymbol{e} = (1, 0, 0)$, and magnetic particles are perpendicular to the director. The uniform phase is stable for $h < h_F$; at $h = h_F$ the second order transition into distorted phase takes place ($\theta_0 \neq 0$, $\theta_m \neq 0$), which is stable for $h_F \leq h \leq h_S$. At $h = h_S$ the second order transition takes place and the distorted state is replaced by the saturation state. In the saturation state the director and magnetic particles are oriented along the field direction. As it is seen from the curve $\tau(h)$, in the uniform state and in the saturation state the light transmission factor is equal to unity. In the distorted phase the transmission factor initially decreases, reaching the minimum, and then increases. As it is seen from Eq. (19), the dependence $\tau(h)$ is determined by $\theta_0(h)$, and the minimum $\tau = 0.292$ of the transmission factor corresponds to the value $\theta_0 = \pi/4$, which corresponds to $h = 7.115$. At $h = 7.115$ the angle of the director rotation relative to the boundaries is equal to $\theta_m - \theta_0 \approx 43^\circ$.

In Figs. 3–5 the angles $\theta_m(h)$, $\theta_0(h)$, and the transmission factor $\tau(h)$ are shown for different values of coupling energy between the magnetic particles and nematic matrix: $w_p = 0.05$ (Fig. 3), $w_p = 0.1515$ (Fig. 4), and $w_p = 1$ (Fig. 5). As it is shown in [4, 5], the parameter ζ of surface coupling anisotropy influences the Freedericksz transition and has two threshold values ζ^* and ζ^{**} (at $\zeta > \zeta^*$ the transition from the uniform phase into the distorted one is the first order transition, and for $\zeta \geq \zeta^{**}$ the first order transition takes place directly from the uniform state into saturation state). For this reason in Figs. 3–5 the parameter $\zeta = 0.3$ is chosen from the condition $\zeta^* < \zeta < \zeta^{**}$, where $\zeta^* = 0.17$ corresponds to the boundary of the first order transitions between the distorted phase and saturation state [5], and $\zeta^{**} = 1 - (h_F^2 - 2w_p)/(2w_0)$ determines the boundary of the first order transitions between the uniform state and saturation state [5]. In Figs. 3–5 the quantity ζ^{**} takes the values 0.654, 0.655, and 0.661, respectively, and vertical lines correspond to first order transitions.

As it is seen from Figs. 3 and 4, at $\zeta = 0.3$ and $h = h_F$ there is the second order orientational transition from the uniform phase into distorted one. The transition from the distorted phase in the saturation state takes place at $h = h_C$ as the first order transition: at $h = h_C$ the free energies (2) of the distorted phase and saturation state become equal. The transmission coefficient $\tau(h)$ has a jump at $h = h_C$ in contrast to the case $\zeta = 0$.

Numerical calculations of Eqs. (4)–(9) at various values of parameter w_p, describing the coupling energy of liquid crystal molecules with the magnetic particles show that there is the first order transition from uniform phase to the distorted phase (Fig. 5). Thus, for given values of material parameters of a ferronematic the tricritical value of parameter w_p exists: at $w_p \leq w_p^*$ the Freedericksz transition is the second order transition and at $w_p > w_p^*$ it is the first order one. Let us notice that tricritical behavior of a ferronematic as a result of segregation effect has been reported recently in Ref. [17]. Numerical calculations of Eqs. (4)–(9) give $w_p^* = 0.1515$ at $\xi = 10$, $\kappa = 2.2 \cdot 10^{-3}$, and $w_0 = 10$; the tricritical case $w_p = w_p^*$ is shown in Fig. 4. For considered orientational transitions the quantity $\sin^2 \theta_m$ plays the role of the order parameter.

As it is seen from Figs. 2–5, in the distorted phase with the increasing of a magnetic field the intensity of transmitted light through a ferronematic almost three times decreases. It is caused by that with the increase in the magnetic field the quantity $\theta_m - \theta_0$ increases [the curve $\theta_m(h)$ grows more abruptly than the curve $\theta_0(h)$] and ferronematic represents the media with appreciable rotation of a plane of light polarization and with the reduction of the difference $(\theta_m - \theta_0)$ the rotation becomes less and the transmission factor tends to unit.

Conclusion

In the present work the calculation of light transmission factor for a ferronematic layer twisted by an external magnetic field and placed between two parallel polarizers is carried out. Dependencies of light transmission factor on external magnetic field, energy of coupling with the boundaries of a layer, and material parameters of a ferronematic are considered. It is shown that in the distorted phase the ferronematic represents the media with appreciable rotation of a plane of light polarization that is shown in reduction of intensity of light propagation through a ferronematic.

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